and temperature, the pressure of air going through the throat of the carburetor and the incoming air are related as:

\[
\frac{T_{th}}{T_I} = \left( \frac{P_{th}}{P_I} \right)^{(k-1)/k} \tag{14.1}
\]

where \( k \) is the ratio of the specific heat at constant pressure, \( c_p = \frac{dh}{dt} \), to the specific heat for constant volume, \( c_v = \frac{du}{dV} \), \( h \) is enthalpy, and \( u \) [J (kg K)\(^{-1}\)] is the internal energy of the flowing air. In this case, for air, \( k \) is a constant and has a value of 1.4 at 20°C. The subscript “th” in the equations stands for the carburetor throat and the subscript “I” stands for the inlet to the carburetor, where the air velocity is assumed to be negligible. With all the above assumptions, the first law of thermodynamics reduces (for details see Chapter 15) to the following:

\[
h_I = h_{th} + \frac{V_{th}^2}{2} \tag{14.2}
\]

where \( h = u + \frac{P}{\rho} = u + R \times T \) is defined as the enthalpy of air, which is the sum of its internal energy and the specific gas constant \( R \) for air times its temperature. For a perfect gas, \( R = c_p - c_v \) and for air, \( R = 287.01 \text{ m}^2 \text{ (s}^2 \text{ K})^{-1} \). Using Eq. (14.2) and the definition of specific heat at constant pressure, the velocity \( V_{th} \) (m s\(^{-1}\)) at the carburetor throat can be formulated as follows:

\[
V_{th} = \sqrt{2 \times c_p \times (T_I - T_{th})} \tag{14.3}
\]

The air mass flow rate \( \dot{m}_{\text{air}} \) (kg s\(^{-1}\)) is defined as:

\[
\dot{m}_{\text{air}} = C_{th} \times \rho_{th} \times A_{th} \times V_{th} \tag{14.4}
\]

where \( C_{th} \) is the coefficient of discharge for the carburetor’s throat area, namely a correction factor between the theory and the real air mass flow rate through the throat. The coefficient of discharge depends on the size, shape, and friction encountered in the throat, and is assumed to have a value of 0.91 in the present analysis. \( \rho_{th} \) (kg m\(^{-3}\)) is the density of air in the throat and \( A_T \) (m\(^2\)) is the area of the throat. The equation of state for an ideal air flow can be written, between the entry to the carburetor and its throat, as:

\[
\frac{P_I}{\rho_I \times T_I} = \frac{P_{th}}{\rho_{th} \times T_{th}} \tag{14.5}
\]

Combining Eqs. (14.1)–(14.5) provides the air mass flow rate through a simple carburetor:

\[
\dot{m}_{\text{air}} = C_{th} \times A_{th} \times \frac{P_I}{\sqrt{R \times T_I}} \times \left( \frac{P_{th}}{P_I} \right)^{1/k} \times \sqrt{2 \times \left( \frac{k}{k-1} \right) \times \left[ 1 - \left( \frac{P_{th}}{P_I} \right)^{(k-1)/k} \right]} \tag{14.6}
\]
For compressible flows, the Mach number is defined as the ratio of the velocity to the local speed of sound. For the velocity of air at the carburetor’s throat, the Mach number is \( M = V_{th} / \sqrt{k \times R \times T_{th}} \). Now, the air mass flow rate in Eq. (14.6) can be written as a function of the throat Mach number:

\[
\dot{m}_{\text{air}} = C_{th} \times A_{th} \times P_{I} \times \sqrt{\frac{k}{R \times T_{I}}} \times M \times \left( \frac{1}{1 + 0.5 \times (k - 1) \times M^{2}} \right)^{(k + 1)/[2 \times (k - 1)]}
\]  \hspace{1cm} (14.7)

The fuel mass flow rate \( \dot{m}_{\text{fuel}} \) entering the throat area can be treated as an incompressible fluid using Bernoulli’s equation. Assuming that the fuel level in the float chamber always stays the same at the narrowest throat location, and the fuel exits into the carburetor from the fuel nozzle end that is \( z \) cm above the narrowest throat location, Eq. (14.8) shows the applicable mechanical energy relationship:

\[
\left( \rho_{\text{fuel}} \times \frac{V_{fuel}^{2}}{2} \right) + P_{th} + \rho_{\text{fuel}} \times g \times z = P_{I}
\]  \hspace{1cm} (14.8)

The velocity of the fuel entering from the throat area into the fuel nozzle can be obtained as:

\[
V_{fuel} = \sqrt{2 \times \left( \frac{P_{I} - P_{th}}{\rho_{\text{fuel}}} - g \times z \right)}
\]  \hspace{1cm} (14.9)

Using Eq. (14.9), the fuel mass flow rate is defined as:

\[
\dot{m}_{\text{fuel}} = C_{\text{nozzle}} \times A_{\text{nozzle}} \times \sqrt{2 \times \rho_{\text{fuel}}(P_{I} - P_{th} - \rho_{\text{fuel}} \times g \times z)}
\]  \hspace{1cm} (14.10)

Equation (14.10) can be rewritten using the Mach number definition of \( \frac{P_{th}}{P_{I}} \) (see Chapter 15), and Eq. (14.10) takes the following form:

\[
\dot{m}_{\text{fuel}} = C_{\text{nozzle}} \times A_{\text{nozzle}} \times \sqrt{2 \times \rho_{\text{fuel}} \times P_{I} \times \left( 1 - \frac{P_{th}}{P_{I}} \right)} \times \frac{\rho_{\text{fuel}} \times g \times z}{P_{I}}
\]  \hspace{1cm} (14.11)

where \( C_{\text{nozzle}} \) is the coefficient of discharge at the fuel nozzle, namely a correction factor between the theory and the real fuel mass flow rate through the nozzle from the throat, \( \frac{P_{th}}{P_{I}} = [1 + 0.5 \times (k - 1) \times M^{2}]^{k/(1-k)} \). The coefficient of discharge for the fuel nozzle depends on the size, shape, and friction encountered in the nozzle, and is assumed to have a value of 0.74 in the present analysis.

Now we can analyze the air/fuel ratio (A/F) behavior in a simple carburetor by using Eqs. (14.7) and (14.11) with the following input parameters:

\[ T_{I} = 293 \text{ K}, \quad P_{I} = 101325 \text{ N m}^{-2}, \quad \frac{A_{th}}{A_{\text{nozzle}}} = 100, 300, 500 \]
Figure 14.1 shows the air/fuel ratio as a function of Mach number for three different throat area-to-fuel nozzle area ratios. For this simple carburetor, $A_{th}/A_{nozzle} = 300$ is very close to the ideal air fuel ratio of 14.7. As $A_{th}/A_{nozzle}$ increases, the air mass flow rate becomes excessive and the air/fuel mixture becomes too lean to burn. As $A_{th}/A_{nozzle}$ decreases, the air mass flow rate becomes depleted and the air/fuel mixture becomes too rich to burn. The A/F ratio is very sensitive to Mach number variations at small Mach numbers (i.e. $M < 0.1$). The A/F mixture becomes leaner as the Mach number increases towards the choking value of unity.

Figure 14.2 presents the sensitivity of the air/fuel ratio to Mach number. The absolute value of this sensitivity decreases at small Mach numbers, goes through a minimum around $M = 0.2$, and starts to increase gradually until its choke value.

Figure 14.3 shows the sensitivity of $P_{th}/P_I$ as a function of the Mach number. As the Mach number increases, the absolute value of the sensitivity also increases.
The absolute value of the sensitivity goes through a maximum at $M = 0.79$ and starts to decrease gradually until $P_{th}/P_i$ reaches the value of 0.528 at $M = 1$.

Figure 14.4 shows the variation of A/F with ambient temperature. As the ambient temperature increases, the air flow into the carburetor decreases and the air fuel mixture becomes too lean to burn. The ideal air fuel ratio of 14.7 is shown as a dot in Figure 14.4 at 20°C.

Figure 14.5 shows the sensitivity of A/F to ambient temperature at choked flow. The absolute value of the sensitivity decreases as the ambient temperature increases.

Figure 14.6 presents A/F as a function of throat diameter-to-fuel nozzle diameter ratio under the choked flow condition. A/F increases as a quadratic function of $d_{th}/d_{nozzle}$. At small diameter ratios the air flow is diminished and results in a rich air/fuel mixture. As the diameter ratio increases the air/fuel ratio becomes lean. In this case, a throat diameter-to-fuel nozzle diameter ratio of 20 provides the ideal burn.
Figure 14.5  Sensitivity of A/F to ambient temperature at sea level at choked flow

Figure 14.6  A/F versus ratio of throat diameter to fuel nozzle diameter at choked flow

Table 14.1  Effects of a ±10% change in nominal values of independent variables on air/fuel ratio, A/F

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal value</th>
<th>+10% value @ M = 0.1</th>
<th>−10% value @ M = 0.1</th>
<th>+10% value @ M = 0.5</th>
<th>−10% value @ M = 0.5</th>
<th>+10% value @ M = 1.0</th>
<th>−10% value @ M = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{th})</td>
<td>2.00 cm</td>
<td>21.00</td>
<td>−19.00</td>
<td>21.00</td>
<td>−19.00</td>
<td>21.00</td>
<td>−19.00</td>
</tr>
<tr>
<td>(d_{fuel})</td>
<td>1.155 mm</td>
<td>−17.36</td>
<td>23.46</td>
<td>−17.36</td>
<td>23.46</td>
<td>−17.36</td>
<td>23.46</td>
</tr>
<tr>
<td>(C_{th})</td>
<td>0.91</td>
<td>10.00</td>
<td>−10.00</td>
<td>10.00</td>
<td>−10.00</td>
<td>10.00</td>
<td>−10.00</td>
</tr>
<tr>
<td>(C_{nozzle})</td>
<td>0.74</td>
<td>−9.09</td>
<td>11.11</td>
<td>−9.09</td>
<td>11.11</td>
<td>−9.09</td>
<td>11.11</td>
</tr>
<tr>
<td>(P_i)</td>
<td>101 325 N m(^{-2})</td>
<td>4.34</td>
<td>−4.53</td>
<td>4.86</td>
<td>−5.11</td>
<td>4.87</td>
<td>−5.12</td>
</tr>
<tr>
<td>(\rho_{fuel})</td>
<td>737 kg m(^{-3})</td>
<td>−4.11</td>
<td>4.81</td>
<td>−4.63</td>
<td>5.39</td>
<td>−4.65</td>
<td>5.40</td>
</tr>
<tr>
<td>(z)</td>
<td>1 cm</td>
<td>0.57</td>
<td>−0.56</td>
<td>0.02</td>
<td>−0.02</td>
<td>0.01</td>
<td>−0.01</td>
</tr>
<tr>
<td>(k)</td>
<td>1.4</td>
<td>−0.51</td>
<td>0.63</td>
<td>0.05</td>
<td>−0.05</td>
<td>0.92</td>
<td>−0.98</td>
</tr>
<tr>
<td>(T_i)</td>
<td>20°C</td>
<td>−0.34</td>
<td>0.34</td>
<td>−0.34</td>
<td>0.34</td>
<td>−0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>
When the nominal values of independent variables used in the present analysis are varied by ±10%, the resulting effects on the air/fuel ratio are as presented in Table 14.1. The resulting effects on the air/fuel ratio are shown in descending order, namely the most effective variables being the carburetor throat and fuel nozzle diameters. Next in order of importance of changing variables for A/F are the coefficients of discharge for the carburetor throat and for the fuel nozzle, whose values depend on size, shape, and friction encountered. The third tier of importance of changing variables for A/F are the atmospheric pressure and the fuel density. The least effective changing variables are the height between the narrowest throat location and the fuel exit location into the carburetor from the fuel nozzle, the ratio of the specific heats, and atmospheric temperature.
Ideal Gas Flow in Nozzles and Diffusers

In this chapter we will analyze properties such as temperature, pressure, and density of an ideal gas flowing through nozzles and diffusers. Analyzing the flow of compressible fluids such as air will require us to use the first and second laws of thermodynamics.

The first law of thermodynamics for a steady flow states that the net amount of energy added to a system as heat and/or work should be equal to the difference between the stored energy of the fluid mass leaving the system and the stored energy of the fluid mass entering the system. The first law can be expressed per unit mass of the fluid as, see Ref [5]:

\[ q + w = (h_2 + 0.5 \times V_2^2 + g \times z_2) - (h_1 + 0.5 \times V_1^2 + g \times z_1) \]  (15.1)

where \( q \) is the heat added to the system, \( w \) is the work done on the system, subscript 2 represents the section where the fluid mass leaves the system, subscript 1 represents the section where the fluid mass enters the system, and \( h \) is the enthalpy of the fluid mass, defined as \( h = u + \frac{P}{\rho} \) where \( u \) is the internal energy and \( \frac{P}{\rho} \) is the flow work done on the system by the entering or leaving fluid mass. Next, two terms in Eq. (15.1) represent the kinetic energy and the potential energy of the fluid mass, respectively. Each term has dimension of J kg\(^{-1}\) or equally m\(^2\) s\(^{-2}\).

Let us now find the ideal gas properties with respect to their stagnation point properties at section 2, where the stagnation point, identified here as section \( s \), is defined as the point at which the fluid mass is brought to rest reversibly, without
work, without heat transfer (i.e. adiabatically), and with negligible change in potential energy. Then, Eq. (15.1) reduces to the following:

\[ 0 = h_s - h_1 - 0.5 \times V_1^2 \] (15.2)

For an ideal gas, stagnation enthalpy and stagnation temperature are related as:

\[ h_s - h_1 = c_p \times (T_s - T_1) \] (15.3)

where \( c_p \) is a thermodynamic property of a substance defined as the specific heat at constant pressure, \( c_p = \left( \frac{\partial h}{\partial T} \right)_p \) [J (kg K)^{-1}]. Substituting Eq. (15.3) into Eq. (15.2) gives the following relationship between the ideal gas flow temperature in a nozzle or diffuser and the stagnation temperature for an ideal gas at rest:

\[ \frac{T_1}{T_s} = \left( 1 + 0.5 \times \frac{V_1^2}{c_p \times T_1} \right)^{-1} \] (15.4)

In compressible fluid mechanics, it is convenient to compare the fluid velocity to the local speed of sound in the fluid by defining the Mach number \( M \) as \( M = \frac{V}{V_s} \). For ideal gases in a reversible and adiabatic flow, also called an isentropic flow, the local speed of sound is defined as \( V_s = \sqrt{k \times R \times T} \) (see Ref. [6], chapter 15). \( k \) is defined as the ratio of specific heats, namely \( k = \frac{c_p}{c_v} \), and the thermodynamic property of a substance defined as the specific heat at constant volume, \( c_v \), is defined as \( c_v = \left( \frac{\partial u}{\partial T} \right)_v \). Then, Eq. (15.4) can be rewritten as:

\[ \frac{T_1}{T_s} = [1 + 0.5 \times (k - 1) \times M^2]^{-1} \] (15.5)

For ideal gases with constant specific heats and for an isentropic process, it can be shown that \( \frac{P_s}{\rho_s^2} = \frac{P_1}{\rho_1^2} \) (see Ref. [6], chapter 15). Then, the ideal gas properties with constant specific heats, for an isentropic process, can be related as:

\[ \frac{T_1}{T_s} = \left( \frac{P_1}{P_s} \right)^{(k-1)/k} = \left( \frac{\rho_1}{\rho_s} \right)^{k-1} \] (15.6)

Using Eqs. (15.5) and (15.6), we can obtain a relationship between the ideal gas flow pressure in a nozzle or diffuser and the stagnation pressure for an ideal gas at rest:

\[ \frac{P_s}{P_1} = [1 + 0.5 \times (k - 1) \times M^2]^{k/(1-k)} \] (15.7)

Using Eqs. (15.5) and (15.6), we can obtain a relationship between the ideal gas flow density in a nozzle or diffuser and the stagnation density for an ideal gas at rest:

\[ \frac{\rho_1}{\rho_s} = [1 + 0.5 \times (k - 1) \times M^2]^{1/(1-k)} \] (15.8)
Using Eqs. (15.4), (15.5), and (15.8), the mass flow rate per unit area, \( \dot{m}_1/A_1 \), through a nozzle or diffuser can be obtained as:

\[
\dot{m}_1/A_1 = \rho_1 \times V_1 \times \sqrt{\frac{k}{R \times T_s}} \times \left[ 1 + 0.5 \times (k - 1) \times M^2 \right]^{(k+1)/(2(k-1)}}
\]

(15.9)

For an ideal gas flow through a nozzle and/or diffuser, the maximum mass flow rate always occurs at \( M = 1 \). This maximum mass flow rate limiting condition is also called a sonic choke. For an ideal air flow with the following input parameters, the mass flow rate per unit area as a function of Mach number is shown in Figure 15.1.

Ideal air input parameters:

\( k = 1, \quad R_{\text{air}} = 286.9 \text{ m}^2(\text{s}^2 \text{ K})^{-1} \)
\( P_s = 101325 \text{ N m}^{-2}, \quad T_s = 293 \text{ K} \)

For the maximum mass flow rate, Eq. (15.9) reduces to the following Eq. (15.10) where the mass flow area is at a minimum at \( M = 1 \):

\[
\left( \frac{\dot{m}_1}{A_{\text{min}}}_\text{max} \right) = P_s \times \sqrt{\frac{k}{R \times T_s}} \times [0.5 \times (k + 1)]^{(k+1)/(2(k-1)}}
\]

(15.10)

By combining Eqs. (15.9) and (15.10), the mass flow circular area diameter ratio \( d_1/d_{\text{min}} \) can be obtained as:

\[
\frac{d_1}{d_{\text{min}}} = M^{-0.5} \times \left[ \frac{1 + 0.5 \times (k - 1) \times M^2}{0.5 \times (k + 1)} \right]^{(k+1)/(4(k-1))}
\]

(15.11)

Figure 15.2 shows \( d_1/d_{\text{min}} \) versus Mach number for an air flow. In a subsonic nozzle \( M < 1 \), the nozzle diameter starts to decrease to the minimum diameter as the
Mach number increases from zero to one. Above Mach number one, namely in a supersonic nozzle, the nozzle diameter increases as the Mach number increases. In a diffuser, the opposite flow phenomenon occurs with respect to the nozzle, namely in a subsonic diffuser, the diffuser diameter increases as the Mach number decreases and in a supersonic diffuser \((M > 1)\), the diffuser diameter decreases as the Mach number increases.

The sensitivity of the circular area diameter to Mach number is presented in Figure 15.3. In the sonic region of a nozzle, the diameter decreases fast as the Mach number increases. The diameter decrease slows down as \(M = 1\) is approached. At \(M = 1\), the sensitivity goes through zero and starts to increase linearly in the supersonic region.

The ideal air properties such as temperature, pressure, and density normalized to the stagnation conditions given above are presented as isentropic flow versus Mach number in Figure 15.4. All the properties decrease as the Mach number increases, both in the sonic region and in the supersonic region for a nozzle. The decrease